

Derivatives of Lipschitz functions

Aris Daniilidis

VADOR, TU Wien, Austria

Abstract

We focus on the difference between differentiable versus strict differentiable locally Lipschitz functions from the view point of nonsmooth analysis: while in the latter class, all limiting Jacobians are singletons, we show that for some locally Lipschitz functions (which, in addition, are everywhere differentiable) the limiting Jacobian map contains all nonempty compact connected subsets of matrices. In the particular case of real-valued functions, we obtain locally Lipschitz functions with surjective limiting and Clarke subdifferentials. Our concrete example-scheme will also reveal that the class of such pathological locally Lipschitz (and everywhere differentiable) functions is dense (for the topology of the uniform convergence) and spaceable (for the Lip-norm topology). A similar omnipresent pathology also prevails subdifferentially saturated functions.

The talk is based on the works:

A. DANIILIDIS, G. FLORES

Linear structure of functions with maximal Clarke subdifferential
SIAM J. Optim. **29** (2019), 511–521.

A. DANIILIDIS, R. DEVILLE, S. TAPIA-GARCIA

All convex bodies are in the subdifferential of some everywhere differentiable locally Lipschitz function, *Proc. London Math. Society* (3) **129** (2024), no. 5, Paper No. e70007, 27 pp., DOI <http://dx.doi.org/10.1112/plms.70007>

A. DANIILIDIS, S. TAPIA-GARCIA

Differentiable functions with surjective Clarke Jacobians
ha1-04952836 (Preprint 15p, 2025)